Catastrophic Inflation

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SUSY 2011

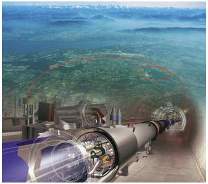
arXiv:1106.2266 , work in progress
Sean Downes, Bhaskar Dutta, KS

Why am I talking about inflation at SUSY 2011?

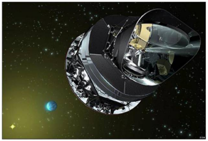
A matching of scales?

We don’t know the scale of inflation and the scale of SUSY breaking

Large Hadron Collider



Planck



Inflation generates metric perturbations: Scalar and Tensor

The scale of inflation is related to the tensor to scalar ratio r through

$$V^{1/4} \sim \left(\frac{r}{0.07}\right)^{1/4} \times 10^{16} \text{ GeV}$$

Planck will get to $r = 0.05$. Gravity waves $\Rightarrow$ inflation at the GUT scale

But what if not?

Inflationary sector has vacuum energy $\Rightarrow$ SUSY broken $\longrightarrow$ it *is* the SUSY breaking of the world

Dine Riotto hep-ph/9705386 , Guth Randall hep-ph/9512439

The Kallosh-Linde problem in String Theory.

Comes from a simple fact at the heart of string theory

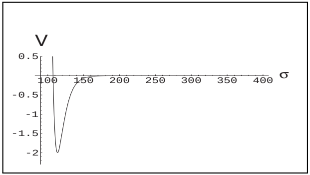
There are extra dimensions of space, and these dimensions are compact

Kallosh, Linde 2004, 2007

Consider KKLT

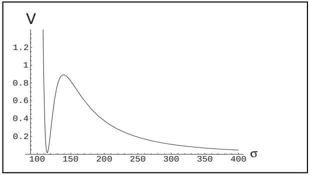
$$K = -3\ln(T + \overline{T})$$

$$W = W_{\rm flux} + A\, e^{-aT}$$



$$V = e^K\left(|DW|^2 - 3W^2\right) + V_{\rm lift}$$

$$m_{3/2}^2 = e^K\, W^2$$

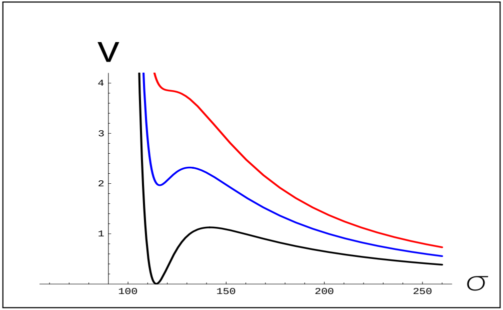


Barrier height $\sim 3m_{3/2}^2$

Consider an inflationary sector ϕ

$$V_{\text{total}} = V = e^K \left(|DW|^2 - 3W^2\right) + V_{\text{lift}} + e^K (D_\phi W^2)$$

$$\sim V = e^K \left(|DW|^2 - 3W^2\right) + V_{\text{lift}} + \frac{C}{\sigma^3}$$



Inflationary scale $\sim$ SUSY breaking scale

Presumably, we should be studying low-scale inflation

$$V^{1/4} \sim \left(\frac{r}{0.07}\right)^{1/4} \times 10^{16}\ \mathrm{GeV}$$

$$\left(\frac{r}{0.07}\right)^{1/2} \lesssim \frac{\Delta\phi}{M_{\mathrm{pl}}} \quad \text{(Lyth bound)}$$

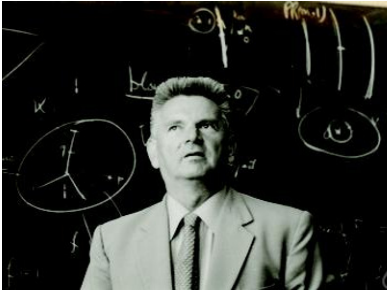
Small-field inflation models

- Natural in the context of low-scale inflation
- Effective action under control

We'll mainly talk about Inflection Point Inflation

Rest of talk:

Catastrophe theory: the mathematics of critical points of functions



Rene Thom

Inflection point inflation:

- Common structure: D-brane inflation, MSSM inflation, Kahler moduli inflation etc.
- $\epsilon,\ \eta \ \ll 1 \Rightarrow V'(\phi_0), V''(\phi_0) \ll 1$
- Relevant data: Inflaton fields $\times$ Space of physical control parameters

$$\Sigma \times C$$

Singularity theory: degenerate critical points

Hessian: Morse $\bigoplus$ non-Morse (Splitting Lemma)

non-Morse (Σ): $V'(\phi_0) = V''(\phi_0) = 0$.

Thom Classification Theorem:

- Classification of all possible $\Sigma \times C$
- For a given inflationary scenario, complete analytic control over control parameter space C

ADE classification of inflaton potentials

$$\Sigma \times C \quad \text{(Thom Classification Theorem)}$$

$A_{\pm k}$: $(\pm)^k x^{k+1} + \Sigma_{m=1}^{k-1} a_m x^m$

$D_{\pm k}$: $(\pm)^k xy^2 \pm x^{2k-1} + \Sigma_{m=1}^{k-3} a_m x^m + c_1 y + c_2 y^2$

$E_{\pm 6}$: $\pm(x^4 + y^3) + ax^2y + bx^2 + cxy + dx + fy$

E_7 : $y^3 + yx^4 + \Sigma_{m=1}^{4} a_m x^m + by + cxy$

E_8 : $x^5 + y^3 + y\Sigma_{m=0}^{3} a_m x^m + \Sigma_{m=1}^{3} c_m x^m$

Information about control parameters space C

Consider A_k singularities

Σ is one-dimensional (single-field inflation)

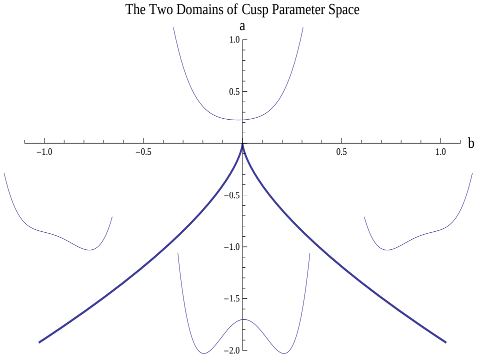
$$V'(x) = v(x) \prod_i (x - \beta_i)$$

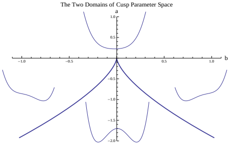
$\beta_1 = \ldots = \beta_m \;\Rightarrow\; (k - m)$ dimensional hypersurface in C.

We will take $m = 2$

A_3 domain structure

$$V(x) = x^4 + \frac{1}{2}ax^2 + bx$$





$$N = \left(\frac{\pi}{2}\right) \frac{1}{2\sqrt{\lambda_1(\beta-\alpha)}}$$

$$\Delta_{\mathcal{R}}^2 = V_0 \frac{N^4}{144\pi^2}(\beta-\alpha)^6$$

- Exactly on the cusp $N \to \infty$
- λ_1 parametrizes how far you go from the cusp
- Can get the probability of having N e-foldings (work in progress)

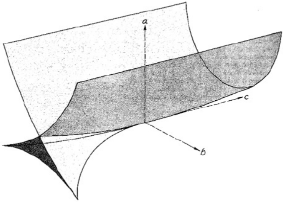
Existence properties:

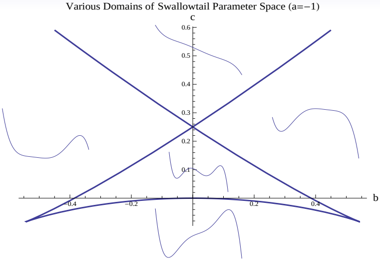
- Inflation happens near domain walls in C
- How close you are depends on how much N you want
- Existence: if physical parameters do not exclude a domain wall, inflation is in principle possible irrespective of (perhaps uncontrolled) corrections

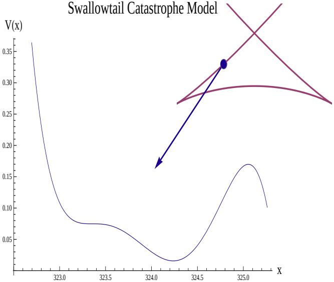
A_4 domain structure

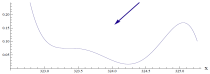
$$V(x) = x^5 + \frac{a}{3}x^3 + \frac{b}{2}x^2 + cx$$

Swallowtail Potential




Various Domains of Swallowtail Parameter Space (a=−1)
c
b
0.6
0.5
0.4
0.3
0.2
0.1
−0.4
−0.2
0.2
0.4


Swallowtail Catastrophe Model
V(x)
x
0.35
0.30
0.25
0.20
0.15
0.10
0.05
323.0
323.5
324.0
324.5
325.0



$$V_{\text{inf}} \propto (\beta-\alpha)^4(\gamma-\alpha)$$

$$V_{\text{barrier}} \propto (\gamma-\alpha)^4(\beta-\alpha)$$

Separation of scales: forced into Large Volume Scenarios?
Dissipation into background radiation?

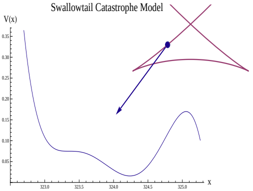
Conlon, Kallosh, Linde, Quevedo 2008

A_4 example: Type IIB racetrack

$$K = -3 \ln (T + \overline{T}), \quad W = W_0 + A\, e^{-aT} + B\, e^{-bT}$$
$$V_{\mathrm{uplift}} = C/(\mathrm{Re}\, T)^2$$

Control parameters

$$(W_0, A, B, C) \longrightarrow (1, \overline{A}, \overline{B}, \overline{C}) = (1, \frac{A}{W_0}, \frac{B}{W_0}, \frac{C}{W_0})$$

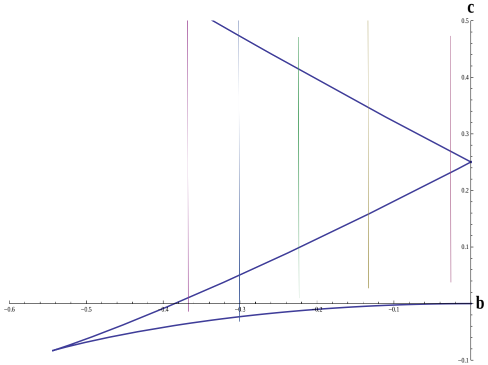


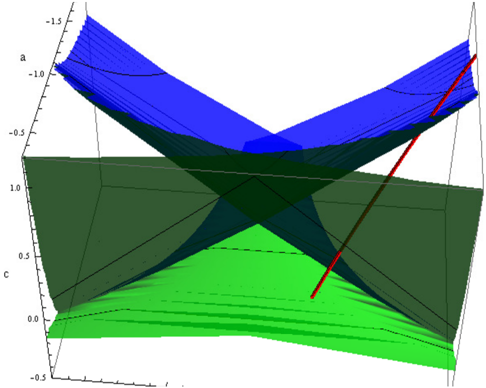
Three parameters and two minima $\longrightarrow$ A_4 inflation

$a \propto \overline{A}, b \propto (\overline{C} - \frac{\overline{B}}{\overline{A}})$

$c \propto (\overline{C} + \frac{\overline{B}}{\overline{A}})$

Curves of Constant B in Swallowtail Control Space




a
-1.5
-1.0
-0.5
c
1.0
0.5
0.0
-0.5

$$\Delta_R^2 \propto (\beta-\alpha)^6(\gamma-\alpha)^3\alpha^6, \quad \alpha \sim \log\left|\frac{A}{W_0}\right|$$

For $\Delta_R^2 \sim 10^{-10}$, $N \sim 50$, intermediate scale inflation, need $\alpha \sim \mathcal{O}(10^2 - 10^3)$. $W_0 \sim 10^{-14} \Rightarrow A \sim e^{\kappa}, \kappa \sim 100$

$$\mathrm{M}_0 = \frac{8\pi|\Delta_{\mathcal{R}}|\alpha^3}{3N_e^2|(\beta-\alpha)(\gamma-\alpha)|(\beta-\kappa)}$$

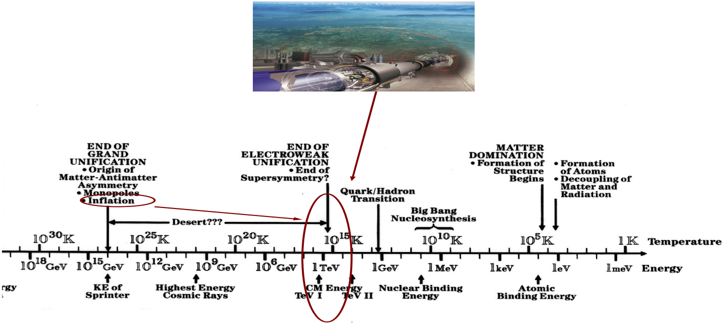
$$\alpha_{\mathrm{mir}} = \frac{\beta-\kappa}{32}$$

Allahverdi, Dutta, KS (arXiv:0912.2324)

A singularity theoretic approach to inflation

- Neat classification of inflation potentials and analytic control over parameter spaces
- Suited for embedding inflationary regions in a larger physical theory
- Stability and universality properties clearer

Applied A_4 singularities to study a complicated inflaton potential in string theory. Found the effect of low scale inflation on supersymmetry breaking in a toy racetrack model



Future directions:

- Explore D and $E-$type singularities, parameter space of multifield inflationary models
- For $A-$type singularities, probe connections between inflation and supersymmetry breaking in more detailed models